Interfering with decay of a single photon in microwave cavities through single photon quantum non-demolition scheme

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The decay of a single photon state in a microwave cavity is shown to be retarded by interaction with a resonant two-level atom in the experimental setup recently developed by Nogues *et al.* [see NOGUES G., RAUSCHENBEUTEL A., OSNAGHI S., BRUNE M., RAIMOND J. M. and HAROCHE S., *Nature*, **400** (1999) 239]. The effect may be interpreted in terms of the temporary removal of the photon from the cavity thereby protecting it from the effects of the environment to which the cavity is coupled. Realistic parameters lead to a significative increase of the survival probability of the photon subsequently to the monitoring interaction.

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A new non-destructive scheme to measure the presence of a single photon in QED cavity (SP-QND, for single photon quantum non-demolition scheme) has been recently implemented and described [1] by Nogues *et al.*. This scheme differs from the previously available procedure [2,3], using dispersive coupling of the cavity mode to an atomic probe, in that the probe atom interacts resonantly with the relevant mode, the interaction time being adjusted so that the system executes a full Rabi cycle while in contact. In this way the atom ends up in its initial state after a full coherent cycle of photon absorption and re-emission. This clearly violates one of the basic requirements for a "non-demolition" measurement of the photon number, namely that the measured quantity should not be affected by the measurement. In this experiment, however, the changes in photon number are reversible, so that the final state contains ideally the same photon number as the initial state.

Here we explore a rather subtle effect of the coherent cycle of photon absorption and re-emission on the decay of the initial single photon excitation. This decay is due to the unavoidable coupling of the cavity mode to its environment, and can be infered e.g. from the change in the probability of having the one photon state at two different times. This can be done, in principle, using the scheme of Nogues *et al.*. As shown below, if an additional atom executes a full Rabi cycle in the time lapse between these two measurements, the latter probability is predicted to increase relatively to the value it would have in the absence of this extra interaction. A simple interpretation of this result follows if we note that during the resonant atom-field interaction the photon is actually absorbed by the atom and is thereby decoupled from the decay process due to the coupling of the relevant mode to its environment. Since the free lifetime of the atomic excitation is much larger than the photon lifetime in the cavity and the transit time of the atom, the effects of environment degrees of freedom on the atomic excitation can in practice be ignored.

Following ref. [1], we consider three Rydberg circular levels i, g and e of the probe atom. The Ramsey zones are tuned to the frequency of the $i \leftrightarrow g$ transition, while the high-Q cavity is tuned to the (sufficiently different) frequency of the $g \leftrightarrow e$ transition. The dressed dynamics of the atom in the leaky high-Q cavity is described in terms of the master equation, written in the interaction picture,

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$$\frac{d}{dt}\widehat{\rho}(t) = -i\frac{\Omega}{2}\left[\widehat{a}^{\dagger}\widehat{\sigma}_{-} + \widehat{a}\widehat{\sigma}_{+}, \widehat{\rho}(t)\right] \\ +k\left[2\widehat{a}\widehat{\rho}(t)\widehat{a}^{\dagger} - \widehat{a}^{\dagger}\widehat{a}\widehat{\rho}(t) - \widehat{\rho}(t)\widehat{a}^{\dagger}\widehat{a}\right], \tag{1}$$

where $\widehat{\rho}$ is the reduced density operator of the atom plus field system, Ω is the vaccuum Rabi frequency, the $\widehat{\sigma}_i$ are Pauli matrices defined as $\widehat{\sigma}_z \equiv |e\rangle\langle e| - |g\rangle\langle g|$, $\widehat{\sigma}_{-} = \widehat{\sigma}_{+}^{\dagger} \equiv |g\rangle\langle e|$ and k is the damping constant related to the Q-value of the microwave cavity. The reduced density operator can be expanded as

$$\widehat{\rho}(t) = \widehat{\rho}_{ee}(t) \otimes |e\rangle\langle e| + \widehat{\rho}_{gg}(t) \otimes |g\rangle\langle g| + \widehat{\rho}_{ii}(t) \otimes |i\rangle\langle i| \\ + \left[\widehat{\rho}_{ge}(t) \otimes |g\rangle\langle e| + \widehat{\rho}_{ie}(t) \otimes |i\rangle\langle e| + \widehat{\rho}_{ig}(t) \otimes |i\rangle\langle g| + \text{h. c.}\right], \tag{2}$$

A rather unwieldy set of coupled equations for the operators $\widehat{\rho}_{ee}(t) \equiv \langle e|\widehat{\rho}(t)|e\rangle$, etc, can be obtained from eq. (1). They can however be handled by solving the equations of motion for their matrix elements in the restricted subspace of zero and one excitations. As an example, the matrix element $\rho_{e0,e0}(t) \equiv \langle 0|\widehat{\rho}_{ee}(t)|0\rangle = \langle e0|\widehat{\rho}(t)|e0\rangle$ obeys the equation of motion

$$\frac{d}{dt}\rho_{e0,e0}(t) = -i\frac{\Omega}{2}\left[\rho_{g1,e0}(t) - \rho_{e0,g1}(t)\right] + 2k\rho_{e1,e1}(t),$$

which is coupled to the equations of motion for $\rho_{g1,e0}$, $\rho_{e0,g1}$ and $\rho_{e1,e1}$. Since the state $e1$ contains two excitations, the last term in the above equation is neglected. A straightforward formal solution of the resulting equations leads to, using again the matrix element $\rho_{e0,e0}(t)$ as an example,

$$\rho_{e0,e0}(t) = e^{-kt}\left[\alpha(t) + \beta(t)\right],$$

where $\alpha(t)$ and $\beta(t)$ are given by

$$\alpha(t) = \frac{1}{2}\left[\rho_{e0,e0}(0) + \rho_{g1,g1}(0)\right] + \frac{k}{\Gamma}\left[\beta_1\left(e^{\Gamma t} - 1\right) - \beta_2\left(e^{-\Gamma t} - 1\right)\right] \tag{3}$$

and

$$\beta(t) = \beta_1 e^{\Gamma t} + \beta_2 e^{-\Gamma t}. \tag{4}$$

The constants β_1 and β_2 are given by

$$\beta_1 = \frac{1}{4\Gamma}\left[(\Gamma + k)\rho_{e0,e0}(0) + (k - \Gamma)\rho_{g1,g1}(0)\right] + \frac{i\Omega}{4\Gamma}\left[\rho_{e0,g1}(0) - \rho_{g1,e0}(0)\right]$$

and

$$\beta_2 = \frac{1}{4\Gamma}\left[(\Gamma - k)\rho_{e0,e0}(0) - (k + \Gamma)\rho_{g1,g1}(0)\right] - \frac{i\Omega}{4\Gamma}\left[\rho_{e0,g1}(0) - \rho_{g1,e0}(0)\right].$$

The parameter Γ appearing in these expressions is formally given as $\Gamma = \sqrt{k^2 - \Omega^2}$. In the experimentally relevant regime of sub-critical damping ($k \sim 10^3\text{s}^{-1}$ and $\Omega/2\pi \sim 50\text{kHz}$) we use $\Gamma = i\sqrt{\Omega^2 - k^2} \equiv i\Omega'$. Thus the frequency with wich the atom and the field exchange excitations is shifted by the damping to a value Ω' slightly lower than Ω. For the initial state

$$\widehat{\rho}(0) = |g\rangle\langle g| \otimes |1\rangle\langle 1|,$$

the matrix element $\rho_{e0,e0}(t)$ appears as

$$\rho_{e0,e0}(t) = \frac{e^{-kt}}{2}\left\{1 - \frac{1}{\Omega'^2}\left[\left(k^2 + \Omega'^2\right)\cos\Omega' t - k^2\right]\right\}.$$

After an effective interaction time $t_i = 2\pi/\Omega'$, the probability of finding the atom in the state e vanishes.

Monitoring the state of the field. Let an atom prepared in state g be sent through the apparatus consisting of the cavity, tuned to the $g \leftrightarrow e$ transition, set between a pair of Ramsey zones tuned to the $i \leftrightarrow g$ transition so that the effect of each of them on the state of the atom is

$$\begin{aligned} |g\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|i\rangle + |g\rangle\right), \\ |i\rangle &\rightarrow \frac{1}{\sqrt{2}}\left(|i\rangle - |g\rangle\right). \end{aligned} \tag{5}$$

Assuming that the initial state of the field in the high-Q cavity is given by

$$\widehat{\rho}_F(0) = P_1 |1\rangle \langle 1| + P_0 |0\rangle \langle 0|,$$

with $P_1 + P_0 = 1$, the initial state of the composite system will be

$$\widehat{\rho}(0) = \frac{1}{2}\left(|i\rangle + |g\rangle\right)\left(\langle i| + \langle g|\right) \otimes \left(P_1 |1\rangle \langle 1| + P_0 |0\rangle \langle 0|\right).$$

This initial condition gives for the relevant matrix elements

$$\rho_{g1,g1}(t) = \frac{P_1}{4} e^{-kt} \left\{1 - \frac{1}{\Omega'^2}\left[\left(k^2 - \Omega'^2\right)\cos\Omega' t + 2\Omega' k \sin\Omega' t - k^2\right]\right\},$$

$$\rho_{e0,e0}(t) = \frac{P_1}{4} e^{-kt} \left\{1 - \frac{1}{\Omega'^2}\left[\left(k^2 + \Omega'^2\right)\cos\Omega' t - k^2\right]\right\},$$

$$\rho_{e0,g1}(t) = -i\frac{\Omega P_1}{4\Omega'^2} e^{-kt}\left(k\cos\Omega' t + \Omega' \sin\Omega' t - k\right),$$

$$\rho_{g0,g0}(t) = \frac{1}{2}\left\{1 - P_1 e^{-kt}\left[1 + \frac{k}{\Omega'^2}\left(k - k\cos\Omega' t - \Omega' \sin\Omega' t\right)\right]\right\},$$

$$\rho_{e0,i1}(t) = -i\frac{P_1}{2} e^{-3kt/2} \sin\left(\frac{\Omega'}{2} t\right),$$

$$\rho_{g1,i1}(t) = \frac{P_1}{2\Omega'} e^{-3kt/2}\left[\Omega' \cos\left(\frac{\Omega'}{2} t\right) - k \sin\left(\frac{\Omega'}{2} t\right)\right],$$

$$\begin{aligned} \rho_{g0,i0}(t) = {} & \frac{1 - P_1}{2} + i\frac{k}{\Omega'} P_1 \left[\left(k + i\Omega'\right)\frac{e^{-1/2\left(3k + i\Omega'\right)t} - 1}{3k + i\Omega'}\right. \\ & \left. - \left(k - i\Omega'\right)\frac{e^{-1/2\left(3k - i\Omega'\right)t} - 1}{3k - i\Omega'}\right], \end{aligned}$$

$$\rho_{g0,g1}(t) = \rho_{g0,e0}(t) = \rho_{g0,e1}(t) = \rho_{g1,i0}(t) = \rho_{e0,i0}(t) = \rho_{g0,i1}(t) = 0.$$

The time evolution of the operator $\widehat{\rho}_{ii}$ is also needed. It can be obtained algebraically as

$$\widehat{\rho}_{ii}(t) = \frac{1}{2}\left[P_1 e^{-2kt} |1\rangle \langle 1| + \left(1 - P_1 e^{-2kt}\right) |0\rangle \langle 0|\right].$$

When the effective interaction time coincides with a full shifted Rabi cycle $2\pi/\Omega'$ the matrix elements associated with level e vanish. As a result, when the atom leaves the cavity the state of the system will be given by

$$\begin{aligned}\widehat{\rho}\left(2\pi/\Omega'\right) &= \widehat{\rho}_{gg}\left(2\pi/\Omega'\right)\otimes|g\rangle\langle g| + \widehat{\rho}_{ii}\left(2\pi/\Omega'\right)\otimes|i\rangle\langle i| \\ &+ \left[\widehat{\rho}_{ig}\left(2\pi/\Omega'\right)\otimes|i\rangle\langle g| + \text{h. c.}\right].\end{aligned}$$

After the second Ramsey zone, the atomic state will once more be changed according to (5), and the state of the composite system becomes

$$\begin{aligned}\widehat{\rho}'\left(2\pi/\Omega'\right) &= \frac{1}{2}\left[\widehat{\rho}_{ii}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gg}\left(2\pi/\Omega'\right) + \widehat{\rho}_{ig}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gi}\left(2\pi/\Omega'\right)\right]\otimes|i\rangle\langle i| \\ &+ \frac{1}{2}\left[\widehat{\rho}_{ii}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gg}\left(2\pi/\Omega'\right) - \widehat{\rho}_{ig}\left(2\pi/\Omega'\right) - \widehat{\rho}_{gi}\left(2\pi/\Omega'\right)\right]\otimes|g\rangle\langle g| \\ &+ \frac{1}{2}\left[\widehat{\rho}_{gg}\left(2\pi/\Omega'\right) - \widehat{\rho}_{ii}\left(2\pi/\Omega'\right) + \widehat{\rho}_{ig}\left(2\pi/\Omega'\right) - \widehat{\rho}_{gi}\left(2\pi/\Omega'\right)\right]\otimes|i\rangle\langle g| \\ &+ \frac{1}{2}\left[\widehat{\rho}_{gg}\left(2\pi/\Omega'\right) - \widehat{\rho}_{ii}\left(2\pi/\Omega'\right) - \widehat{\rho}_{ig}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gi}\left(2\pi/\Omega'\right)\right]\otimes|g\rangle\langle i|.\end{aligned} \tag{6}$$

At this moment, taking the limit of vanishing dissipation ($k \to 0$), the atomic state $i(g)$ is completely correlated with the 0(1)-photon state of the field in the high-Q cavity. In the subspace spanned by the vectors $\{|0\rangle, |1\rangle\}$, the operadors $\widehat{\rho}_{ii}\left(2\pi/\Omega'\right)$, $\widehat{\rho}_{gg}\left(2\pi/\Omega'\right)$ and $\widehat{\rho}_{ig}\left(2\pi/\Omega'\right) = \widehat{\rho}_{gi}^{\dagger}\left(2\pi/\Omega'\right)$ are given by the matrices

$$\widehat{\rho}_{ii}\left(2\pi/\Omega'\right) = \frac{1}{2}\begin{pmatrix} P_1 e^{-4\pi k/\Omega'} & 0 \\ 0 & 1 - P_1 e^{-4\pi k/\Omega'}\end{pmatrix},$$

$$\widehat{\rho}_{gg}\left(2\pi/\Omega'\right) = \frac{1}{2}\begin{pmatrix} P_1 e^{-2\pi k/\Omega'} & 0 \\ 0 & 1 - P_1 e^{-2\pi k/\Omega'}\end{pmatrix},$$

and

$$\widehat{\rho}_{gi}\left(2\pi/\Omega'\right) = \frac{1}{2}\begin{pmatrix} -P_1 e^{-3\pi k/\Omega'} & 0 \\ 0 & 1 - P_1 + P_1\left(e^{-3\pi k/\Omega'} + 1\right)\frac{8k^2}{9k^2+\Omega'^2}\end{pmatrix}.$$

If the atomic state is *not* measured, the state of the field becomes

$$\begin{aligned}\widehat{\rho}'_F\left(2\pi/\Omega'\right) &= \operatorname{tr}_A \widehat{\rho}'\left(2\pi/\Omega'\right) \\ &= \widehat{\rho}_{ii}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gg}\left(2\pi/\Omega'\right) \\ &= \frac{P_1}{2}\left(e^{-4\pi k/\Omega'} + e^{-2\pi k/\Omega'}\right)|1\rangle\langle 1| \\ &+ \left[1 - \frac{P_1}{2}\left(e^{-4\pi k/\Omega'} + e^{-2\pi k/\Omega'}\right)\right]|0\rangle\langle 0|.\end{aligned}$$

Note that this state is different from the one that would have been obtained if the cavity field had been left to evolve undisturbed. In this case the state of the field would have been

$$\widehat{\rho}_F\left(2\pi/\Omega'\right) = P_1 e^{-4\pi k/\Omega'}|1\rangle\langle 1| + \left(1 - P_1 e^{-4\pi k/\Omega'}\right)|0\rangle\langle 0|.$$

The probability for detecting the atom in state g when the state of the composite system is given by (6) is found to be

$$P_g = \frac{P_1}{2}\left(\frac{k^2+\Omega'^2}{9k^2+\Omega'^2}\right)\left(e^{-3\pi k/\Omega'} + 1\right). \tag{7}$$

If the atom is in fact detected in state g, the state of the field will be reduced to

$$\begin{aligned}\widehat{\rho}_F^{'}\left(2\pi/\Omega', g\right) &= \frac{\langle g|\,\widehat{\rho}^{'}\left(2\pi/\Omega'\right)|g\rangle}{P_g} \\ &= \frac{1}{2P_g}\left[\widehat{\rho}_{ii}\left(2\pi/\Omega'\right) + \widehat{\rho}_{gg}\left(2\pi/\Omega'\right) - \widehat{\rho}_{ig}\left(2\pi/\Omega'\right) - \widehat{\rho}_{gi}\left(2\pi/\Omega'\right)\right] \\ &= P_1\left(g\right)|1\rangle\langle 1| + P_0\left(g\right)|0\rangle\langle 0|\,,\end{aligned}$$

where we defined

$$P_1\left(g\right) \equiv \frac{P_1}{4P_g} e^{-2\pi k/\Omega'}\left(e^{-\pi k/\Omega'} + 1\right)^2$$

with $P_1\left(g\right) + P_0\left(g\right) = 1$.

Changing the field decay. Assume that a one-photon state is created in the microwave cavity and left to evolve there for a short time interval Δt. The state of the field after this time lapse will be

$$\widehat{\rho}_F\left(\Delta t\right) = e^{-2k\Delta t}|1\rangle\langle 1| + \left(1 - e^{-2k\Delta t}\right)|0\rangle\langle 0|\,. \tag{8}$$

Let next an atom be sent through the apparatus to probe the state of the field. Call this atom the "measuring atom". The probability that the measuring atom is detected in state g can be calculated from (7) to be

$$P_g\left(\Delta t\right) = \frac{e^{-2k\Delta t}}{2}\left(\frac{k^2 + \Omega'^2}{9k^2 + \Omega'^2}\right)\left(e^{-3\pi k/\Omega'} + 1\right).$$

Consider, furthermore, the case in which a second atom (to be called the "monitoring atom") be sent through the apparatus *during* the time interval Δt, and let its final state remain undetected. The state of the field after the time interval $\Delta t \geq 2\pi/\Omega'$ will be given, as a result of the monitoring, by

$$\begin{aligned}\widehat{\rho}_F^{(M)}\left(\Delta t\right) &= \frac{1}{2}e^{2\pi k/\Omega'}e^{-2k\Delta t}\left(e^{-2\pi k/\Omega'} + 1\right)|1\rangle\langle 1| \\ &\quad + \left[1 - \frac{1}{2}e^{2\pi k/\Omega'}e^{-2k\Delta t}\left(e^{-2\pi k/\Omega'} + 1\right)\right]|0\rangle\langle 0|\,.\end{aligned}$$

A second measuring atom is next sent in. The probability that it is detected in state g is

$$P_g^{(M)}\left(\Delta t\right) = \frac{P_1^{(M)}\left(\Delta t\right)}{2}\left(\frac{k^2 + \Omega'^2}{9k^2 + \Omega'^2}\right)\left(e^{-3\pi k/\Omega'} + 1\right).$$

Since

$$P_1^{(M)}\left(\Delta t\right) = \frac{1}{2}e^{2\pi k/\Omega'}e^{-2k\Delta t}\left(e^{-2\pi k/\Omega'} + 1\right),$$

we get

$$P_g^{(M)}\left(\Delta t\right) = \frac{1}{4}e^{2\pi k/\Omega'}e^{-2k\Delta t}\left(e^{-2\pi k/\Omega'} + 1\right)\left(\frac{k^2 + \Omega'^2}{9k^2 + \Omega'^2}\right)\left(e^{-3\pi k/\Omega'} + 1\right).$$

Taking the ratio

$$\frac{P_g^{(M)}\left(\Delta t\right)}{P_g\left(\Delta t\right)} = \frac{1}{2}\left(1 + e^{2\pi k/\Omega'}\right), \tag{9}$$

we see that the probability of detecting the second measuring atom in state g is increased as a result of the monitoring process. As already mentioned, this enhancement can be understood as resulting from the temporary removal of the photon from the cavity by the monitoring atom, thus making it unavailable for the decay process. An effective absence time τ can be defined in terms of the ratio (9) as

$$\frac{1}{2}\left(1+e^{2\pi k/\Omega'}\right)=e^{\Omega'\tau/2\pi}$$

so that clearly $0<\tau<2\pi/\Omega'$. If the interaction time $2\pi/\Omega'$ is very short in the scale of the decay time $1/k$, the effect of the monitoring atom on the ratio (9) tends to disappear.

For the experimental values of Ω and k in refs. [1,4], we get an enhancement of 0.5% for the probability $P_g^{(M)}(\Delta t)$. This effect would hardly be detectable. However, for lower-Q cavities, the situation may change drastically: if one uses $10^4\ \mathrm{s}^{-1} \lesssim k \lesssim 10^5\ \mathrm{s}^{-1}$, the renormalized Rabi frequency Ω' remains essentialy unaltered and the ratio (9) varies within a large range:

$$1.1 \lesssim \frac{P_g^{(M)}(\Delta t)}{P_g(\Delta t)} \lesssim 4.$$

Recently, an experiment using a similar scheme was realized at the Ecole Normale Supérieure, in Paris [4]. Although this experiment was intended to measure the Wigner function at the origin of the phase space for single photon and vacuum states, it can be adapted to verify the results obtained here.

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